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An approach for product life functionality under component deterioration process [†]

Chien-Ping Chung and Angus Jeang*

Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung, 407, Taiwan

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Abstract

Product designers are always concerned that a newly developed product is properly performing its functionality for its intended life under consumer usage conditions. It is known that the failure rate is increased in the late time of product life cycle as the consequence of deterioration being built up. Namely, continuous changes may take place in the parameter values of product and the product application may be ended before expiration of its intended life. Hence, a design that considers parameter compensation to extend using time becomes an important factor in earlier stages of product design. Other than the parameter values, the tolerance values are an important element affecting the product performance, which are also needed to be decided appropriately. In this paper, an optimization model considers minimizing the total cost, which includes material cost, inspection cost, quality loss, failure cost, and tolerance cost by conducting concurrent optimization of the decision variables, initial setting, process mean, process tolerance, and using time. The design constraints are the restrictions resulting from process capability limits, functionality requirements, and quality necessities. The software GAMS was used to find optimal values for the decision variables of interest. Finally, an example of various components and subassemblies under deterioration process is presented to explain the proposed model and sensitivity analysis on some decision variables is performed.

Keywords: Quality loss; Tolerance cost; Deterioration; Product life; Optimization

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1. Introduction

At the product/process design stage, the variability of the product quality value can be reduced by two approaches: the first approach uses a parameter design, which adjusts the process mean so that the product quality value is less sensitive to the causes of variability; the other approach is to use a tolerance design that seeks to reduce the process tolerance in order to control the variability of the product quality values [10, 13, 14, 16]. Usually, there is no impact on the cost associated with changing the process mean, whereas reducing the process tolerance leads to an increase in cost. Thus, for economic reasons a process mean determination is usually carried out prior to the tolerance design. However, due to the dependence between process mean and process tolerance, the production of high-quality products at low costs in the current manufacturing industry requires simultaneous consideration of parameter and tolerance design, particularly in the early stages of the product/process design.

As the life of a product influences the assigning of process mean and process tolerance for successful achievement of quality performance during its application, a product designer is always concerned with the life of a newly designed product under consumer usage conditions. A product can usually be designed having a single or multiple components, depending on its intended applications. Thus, the product life is a

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*Corresponding author. +886 4 2451 7250, Fax.: +886 4 2451 0240
E-mail address: akjeang@fcu.edu.tw

function of the life of its components, i.e., failure of the product can lead to its replacement as a whole in the case of a single component product, while one or more components may need to be replaced for an assembled product [2, 17]. Conceptually, before usage of a product, all of its product quality values should be on their design target. However, during product application, the product quality values of components may change, which may gradually diminish the functionality of the product. In the end, the risk of product failure increases and the quality of the product performance decreases [12]. A continuous change in the value of the product quality may take place and the product/component may deteriorate before expiration of its expected life. Consequently, considering process mean for quality compensation and using time for possible deterioration becomes an important factor in the design activities [4,5]. In addition to finding the process mean, determining the tolerance value is also important because it affects the variability of the product quality value during application.

As is known, a change in the process means will result in various quality losses, while an alteration of process tolerances will affect tolerance cost and quality loss at the same time. The tolerance cost includes those costs incurred before a product is delivered to the consumer, quality loss is incurred after a product is delivered. Furthermore, the values of process mean and process tolerance will have a collective impact on the failure cost if the product quality value falls outside the specification limits. Hence, quality loss, failure cost and tolerance cost should be considered simultaneously to appropriately represent the total cost equation introduced in the proposed model. Other than the total cost considered in the proposed model, the product quality should also be included for risk consideration, namely, the product reliability. In sum, the goal of this paper in the proposed model is to minimize the total cost by satisfying the process capability limits, as well as the constraints from the functionality and quality necessities.

This paper is written in five sections. Section 1 is the introduction. Section 2 describes the background review needed in this research. Section 3 presents the problem formulation. Section 4 provides application example and sensitivity analysis. Finally, a conclusion is given in Section 5.

2. Background review

2.1 Process mean deviation and estimation of process variance

The primary objective of quality engineering efforts is the systematic reduction of variability in the product quality values. At the product design stage, the parameter and tolerance design determine the best values of the design target and design tolerance, respectively, so that the non-conforming rate or the variability of the product quality value is reduced to a minimum. Then, at the process design stage, process engineers should specify the process mean and the process tolerance with reference to the design target and the design tolerance obtained at the product design stage so that the manufacturing feasibility can be guaranteed. If the design tolerance is smaller than the process tolerance, no functional product can be produced in the manufacturing process. A manufacturer should aim for a high process capability with a small process tolerance, which always results in a high tolerance cost. However, if the design tolerance exceeds the process tolerance, additional space for the process distribution allows for flexibility within the design tolerance [3, 6]. This flexibility is necessary, as the process mean changes due to process shifting or component deterioration and permits adjustment of the process mean to improve product quality, reduce costs, and increase endurance.

This research concerns a determination of process mean and process tolerance subject to product deterioration as a function of using time. When process mean U is time-dependent due to the deterioration, it can be expressed as a function of time s: $U(s) = a_0 + (B + W \times s)$. Here, a_0 is the initial setting, B is a constant value resulting from initial impact at the beginning of product life, and W is the deterioration rate. Therefore, a functional relationship is provided mathematically in Eq. (1), to link design target T, design tolerances L_1 and L_2, process mean U, and process tolerance t.

$$T - L_1 + t \leq U(s) \leq T + L_2 - t \tag{1}$$

Usually, the existence of a process variance σ^2 is a common feature in all manufacturing processes. In most cases, the process variance is unknown due to unavailability of previous data, particularly for new products or new processes, unless there are previous data from similar processes. Therefore, a reasonable estimation of the process variance is an important issue. As is known, the variance is a function of the process tolerance [7]. If a stable status of the manufacturing process after a long period of production is assumed, in which the process capability index C_P turns to be a constant value, then the product engineers or process engineers can indirectly estimate the process variance in the early stage of the product/process stage through the following relation:

$$\sigma^2 = \left(\frac{t}{3 \times C_P}\right)^2 \tag{2}$$

2.2 Quality loss, failure cost and tolerance cost

Quality loss is an expression that represents the difference between the process mean and the design target, and the variability of the product quality value in terms of economic loss due to product failure in the eyes of a consumer. The main quality loss functions include the nominal-the-best, the smaller-the-better, the larger-the-better and the asymmetric loss function [10,13].

In this paper, the asymmetric loss function is applied. The quality loss experienced by consumers varies in both directions around the design target, i.e., the quality loss resulting from the deviation of the product quality value in one direction is unequal to the quality loss resulting from deviation in the opposite direction. In this case the quality loss coefficient values K_1 and K_2, have to be allocated for the two directions of the design target. We also assume that the quality value of the product delivered to consumers falls within specification limits; the quality value accepted by consumers is between the lower specification limit $T- L_1$ and the upper specification limit $T + L_2$. When the product quality value does not meet specification limits, the product is deemed unfit and should be rejected either to be repaired or to be discarded. If the quality value falls below $T- L_1$, failure cost C_1 will be incurred. On the other hand, if the quality value falls outside $T + L_2$, this will result in failure cost C_2. Thus, the loss function L (X) can be represented as:

$$L(X) = \begin{cases} C_2 \text{ if } T + L_2 \leq X < \infty \\ K_2 (X - T)^2 \text{ if } T \leq X < T + L_2 \\ K_1 (X - T)^2 \text{ if } T - L_1 \leq X < T \\ C_1 \text{ if } 0 \leq X < T - L_1 \end{cases} \tag{3}$$

where X is the quality value and T is the design target.

Usually, a high tolerance cost is associated with a tight process tolerance, while a low tolerance cost results from a loose process tolerance. The tolerance cost can be formulated in various functional expressions. To evaluate the tolerance cost, this paper adopts the tolerance cost function developed in the literature [11].

$$C_M(t) = a + b \times \exp(-c \times t) \tag{4}$$

where a, b, c are the coefficients for the tolerance cost function and t is the process tolerance.

From the above cost expression, it can be noted that a tight process tolerance results in a higher tolerance cost given in Fig. 1; due to additional manufacturing operations, more expensive equipment is needed and slower production rates, while a loose process tolerance results in a lower tolerance cost.

2.3 Process tolerance stack-up, variance build-up and resultant process mean

In practice, the quality values of the completed product result from several single components. A number of analysis methods for resultant tolerance models have been presented, such as the worst case model, the statistical model, the mean shift model and the Monte-Carlo model [18]. In this paper, the worst case model was adopted for problem analysis. Thus, to extend the tolerance design for a completed component, process tolerance stack-up and variance build-up models need to be applied. The process tolerance stack-up and the variance build-up can be estimated by using the chain relationship, where the formulation of the process tolerance stack-up is de-

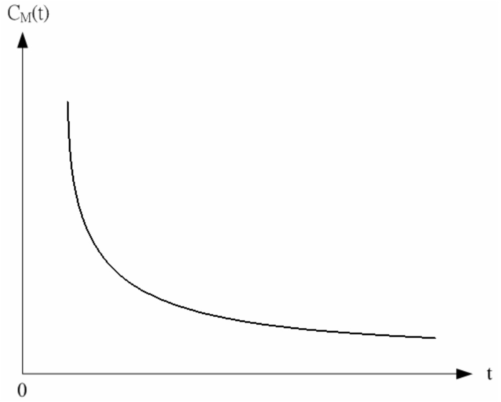

Fig. 1. Tolerance cost function.

rived by accumulating the process tolerance that appears in the chain. Similarly, the process variance build-up possesses the same features, so the process tolerance stack-up and process variance build-up can be represented as:

$$t_{Yr} \approx \sum_{j \in M_{\mathrm{r}}} [(|\frac{\partial f_d}{\partial \mathrm{X}_{j\mathrm{r}}}|_{X_{1\mathrm{r}}=U_{1\mathrm{r}}, X_{2\mathrm{r}}=U_{2\mathrm{r}}, \dots, X_{M_0 r}=\mathrm{U}_{M_0 r}}) \, t_j] \quad (5)$$

$$\sigma_{Yr}^2 \approx \sum_{j \in M_{\mathrm{r}}} [(|\frac{\partial f_d}{\partial \mathrm{X}_{j\mathrm{r}}}|_{X_{1\mathrm{r}}=U_{1\mathrm{r}}, X_{2\mathrm{r}}=U_{2\mathrm{r}}, \dots, X_{M_0 r}=\mathrm{U}_{M_0 r}})^2 \sigma_j^2] \quad (6)$$

where t_j is the process tolerance of the j^th^ component/process. σ_{j}^2 is the process variance of the j^th^ component/process. t_{Yr} is the process tolerance stack-up of the r^th^ product quality value (also called r^th^ resultant process tolerance). M_r is the related components/processes contained in the r^th^ quality chain and are the partial processes from M_0. M_0 is the total number of the components/processes in a product. r is 1, 2, … , M_B. M_B is the total number of quality chains for a product. $\sigma_{\mathrm{Yr}}{}^2$ is the process variance build-up of the r^th^ product quality value (also called r^th^ resultant process variance). X_{jr} is the quality value representing j^th^ component/process quality characteristic which is an element of r^th^ product quality characteristic. U_{jr} is the process mean of the quality characteristic X_{jr}. $f_d(\cdot)$ is a design function representing a functional relationship between the quality value of component/process quality characteristics and the quality values of product quality characteristics. Based on that, a decision to consider process tolerance stack-up and process variance build-up as constraints in the formulation stage of the proposed model is critical.

Because the example introduced in this paper discusses the quality chain vector that involves only the linear equations with coefficients being 1 or –1. Eqs. (5) and (6) can be further simplified into the following expressions:

$$t_{Yr} \approx \sum_{j \in M_r} t_j \quad (7)$$

$$\sigma_{Yr}^2 \approx \sum_{j \in M_r} \sigma_j^2 \quad (8)$$

With the same assumption that the design function forms a linear equation X_{Yr}, the r^th^ resultant quality values for product quality characteristics can be summed up from several single component quality values X_{jr} as the following expression.

$$\sum_{\mathrm{j} \in \mathrm{Mr}} \mathsf{A}_{\mathrm{rj}} \times \mathsf{X}_{\mathrm{jr}} = \mathsf{X}_{\mathrm{Yr}} \quad (9)$$

A_{rj} is the quality chain vector in linkage r^th^ product quality value with j^th^ component/process [8, 9, 15]. By taking the expectation on both sides of Eq. (9), the expected value of the r^th^ resultant product quality value is:

$$\sum_{j \in Mr} \mathsf{A}_{\mathrm{rj}} \times \mathsf{U}_{\mathrm{jr}} = \mathsf{U}_{\mathrm{Yr}} \quad (10)$$

where U_{jr} is process mean of j^th^ component/process quality characteristic for r^th^ product quality characteristic.

3. Problem formulation

Let r^th^ resultant product quality value X_{Yr} fall between the lower specification limit $T_r - L_{1r}$ and the upper specification limit $T_r + L_{2r}$. Then, it is possible that the quality loss experienced by consumers is unequal in both directions around the design target. Therefore, quality loss is evaluated by using the asymmetric loss function with the unequal quality loss coefficients K_{1r} and K_{2r}. On the other hand, the internal (before delivery) failure cost can be estimated as follows: when a product has a quality value that does not meet specification limits, it is rejected either to be repaired or to be discarded. Since the failure cost has distinct values at both directions, the failure cost C_{1r} will be incurred if the quality value falls below $T_r - L_{1r}$, while the failure cost C_{2r} will result if the quality value falls above $T_r + L_{2r}$. Assuming that the resultant product quality value also forms a normal distribution f (X_{Yr}) with a mean that equals the resultant process mean U_{Yr} calculated from Eq. (10) the variance of the normal distribution equals the resultant process variance $\sigma_{\mathrm{Yr}}{}^2$ estimated from Eq. (8). The product quality value X_{Yr} forms a normal distribution with the range from 0 to ∞. The reason for excluding the range from - ∞ to 0 is that the quality value is assumed to be positive in proposed model.

Post-production quality can be either acceptable or unacceptable. An unacceptable product indicates that the quality measurement has fallen outside the design tolerances limits. To illustrate this, the following discussion for rth resultant product quality value from M_B quality chains is divided into two cases.

1. For an acceptable case ($T_r - L_{1r} \leq X_{Yr} \leq T_r + L_{2r}$)

The quality loss of a product is $K_{1r} (X_{Yr} - T_r)^2$ when the product quality value ranges between $T_r - L_{1r}$ and design target T_r, or $K_{2r} (X_{Yr} - T_r)^2$ when the product quality value ranges between design target T_r and $T_r + L_{1r}$. The expected quality loss rth resultant product quality value is:

$$\int_{T_r - L_{1r}}^{T_r} K_{1r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} + \int_{T_r}^{T_r + L_{2r}} K_{2r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} \tag{11}$$

2. For the failure cases ($0 \leq X_r < T_r - L_{1r}$ and $T_r + L_{2r} \leq X < \infty$)

When the quality value falls outside its design specification limit, the defective products contain the failure costs C_{1r} and C_{2r}, respectively. Assuming the product quality value is always positive in this model, the integral interval will exclude the range from $-\infty$ to 0 in the following Eq. (12). The expected failure cost for rth resultant product quality value is:

$$C_{1r} \times \int_{0}^{T_r - L_{1r}} f(X_{Yr})\, dX_{Yr} + C_{2r} \times \int_{T_r + L_{2r}}^{\infty} f(X_{Yr})\, dX_{Yr} \tag{12}$$

The production cost for a product including tolerance cost and inspection cost IC_r for rth product quality characteristics (quality chains) is:

$$\sum_{j=1}^{M_0} [a_j + b_j \times \exp(-c_j \times t_j)] + \sum_{r=1}^{M_B} IC_r \tag{13}$$

For the entire product, other than the above quality loss, failure cost, and production cost, the additional cost such as material cost C_N should be included. Thus, the expected total cost for M_0 processes and M_B product quality characteristics (quality chains) is:

$$\begin{aligned} & C_N + \sum_{j=1}^{M_0} [a_j + b_j \times \exp(-c_j \times t_j)] + \\ & \sum_{r=1}^{M_B} IC_r + \sum_{r=1}^{M_B} [\, C_{1r} \times \int_{0}^{T_r - L_{1r}} f(X_{Yr})\, dX_{Yr} + \\ & \int_{T_r - L_{1r}}^{T_r} K_{1r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} + \\ & \int_{T_r}^{T_r + L_{2r}} K_{2r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} + \\ & C_{2r} \times \int_{T_r + L_{2r}}^{\infty} f(X_{Yr})\, dX_{Yr}] \end{aligned} \tag{14}$$

Let Q be the using time needed to be determined in the proposed model, then, the expected total cost per unit time in duration Q, TC, can be expressed by:

$$\begin{aligned} \mathrm{TC} = \{ & C_N + \sum_{j=1}^{M_0} [a_j + b_j \times \exp(-c_j \times t_j)] + \sum_{r=1}^{M_B} IC_r + \\ & \int_{0}^{Q} \sum_{r=1}^{M_B} [\, C_{1r} \times \int_{0}^{T_r - L_{1r}} f(X_{Yr})\, dX_{Yr} + \\ & \int_{T_r - L_{1r}}^{T_r} K_{1r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} + \\ & \int_{T_r}^{T_r + L_{2r}} K_{2r} \times (X_{Yr} - T_r)^2 \times f(X_{Yr})\, dX_{Yr} + \\ & C_{2r} \times \int_{T_r + L_{2r}}^{\infty} f(X_{Yr})\, dX_{Yr}]\; ds \} \;/\; Q \end{aligned} \tag{15}$$

Most design problems are dictated by design constraints. Design constraints are restrictions of the process capability limits and functionality requirements as well as quality necessities. Thus, in addition to the objective function given in Eq. (15), these constraints must be considered in the problem formulation.

Process capability limits:

$$t_{Lj} \leq t_j \leq t_{Uj} \tag{16}$$

where t_{Uj} and t_{Lj} are upper and lower process capability limits, respectively.

Functionality requirements:

$$T_r - L_{1r} + t_{Yr} \leq U_{Yr} \leq T_r + L_{2r} - t_{Yr} \tag{17}$$

where t_{Yr} and U_{Yr} are defined in Eqs. (7) and (10), respectively.

Quality necessities:

$$R_Y \geq R_{min} \tag{18}$$

where R_Y is defined in Appendix, Eq. (23) and R_{min} is the minimal quality requirement, a given value.

The difference between Eq. (15) and Eq. (18) is that the former one emphasizes the quality value in terms of monetary wise for production management and the latter one is concerned about quality performance in terms of probability wise for consumer application approximately. Thus, Eq. (15) is the estimated economic and quality measurement during the producer's production. Eq. (18) is the estimated quality measurement for the consumer's application. Thus, two equations must appear at the same time in formulating the proposed mode for a life cycle application. Consequently, an economic and quality of applicable product design can be realized.

4. Application example and sensitivity analysis

Assembly is the process by which the various components and subassemblies are brought together to form a completed product designed to fulfill a certain product function. Tolerance design as well as parameter design should be considered simultaneously, particularly when the elements of quality loss, failure cost, inspection cost and tolerance cost are included in the objective function of interest. Therefore, to ensure that the required functionality and quality of a product are met, a proper determination of process mean and process tolerance is of critical importance. In case of deterioration in some components of a product, the additional parameter values that need to be determined are the initial settings and the using time.

The assembly application shown in Fig.2 is a shaft-bearing system of five components: X_{11}, X_{21}, X_{31}, X_{41} and X_{51}. In this example r is 1. As M_1 contains five components, M_0 is 5 and the quality chain vector is $A_{1j} = [1,1,-1,-1,-1]$, the design function in describing the product quality value X_{Y1} of interest is:

$$X_{Y1} = X_{11} + X_{21} - X_{31} - X_{41} - X_{51} \tag{19}$$

Among the five components, components X_{31} and X_{41} are subjected to the deterioration. The deterioration is a function of the time s. Considering deterioration influence, the process means U_{31} and U_{41} of X_{31} and X_{41} can be expressed as $(a_{03} - B_3 - W_3 \times s)$ and $(a_{04} - B_4 - W_4 \times s)$, respectively. Thus, the design function in describing the resultant process mean of the product quality value X_{Y1} of interest is:

$$U_{Y1} = U_{11} + U_{21} - (a_{03} - B_3 - W_3 \times s) - (a_{04} - B_4 - W_4 \times s) - U_{51} \tag{20}$$

Associated process means U_{11}, U_{21}, and U_{51}, initial settings a_{03} and a_{04}, using time Q, as well as process tolerances t_1, t_2, t_3, t_4, and t_5 must be determined so that the product quality value X_{Y1} falls within the specification limits, $T_1 - L_{11}$ and $T_1 + L_{21}$, as possible as design can be, where T_1 is 0.9 mm, L_{11} is 0.16 mm, and L_{21} is 0.20 mm. Let K_{11} and K_{21} be \$3500 and \$2800, respectively. In addition, failure costs are \$5000 and \$3000 for C_{11} and C_{21}, respectively. It is known that the deterioration rate values W_3 and W_4 are 0.020 mm/month and 0.035 mm/month. The coefficients B_3 and B_4 are 0.0000001 and 0.0000001. The capability index, C_P, is assumed to be 1. The quality for X_{Y1} within a given range should be greater than R_{min} which is 0.92 in this example. Table 1 provides the upper and lower process capability limits for each component. Table 2 provides the coefficients a, b, and c for the tolerance cost functions. These can be found based on actual data collected in factories and analyzed through a statistical regression method.

Let the above known values be inserted into Eq. (15), which is treated as the objective function and Eqs. (16), (17), and (18) which are considered as the constraints, then a complete mathematical programming model representing the present problem can be formulated. In our example, the software GAMS was adopted to find the optimal values [1]. These optimal values are: $U_{11}^* = 16.0121$ mm, $U_{21}^* = 18.0150$ mm, $a_{03}^* = 29.0532$ mm, $a_{04}^* = 1.9218$ mm, $U_{51}^* = 2.2913$ mm, $t_1^* = 0.0275$ mm, $t_2^* = 0.0275$ mm, $t_3^* = 0.0436$ mm, $t_4^* = 0.0183$ mm and $t_5^* = 0.0183$ mm, $Q^* = 3.7043$ months, $R_Y^* = 0.9907$, and $TC^* = \$884.7659$/month.

A sensitivity analysis was performed using the deterioration rate W_3 and W_4 values. The results are shown in Table 3, and we have the following conclusion. When the deterioration rate increases, this drives TC^* to increase. An increase in the deterioration rate results in a reduction of the Q^* value and vice versa. This may be explained as follows: when the deterioration rate increases, the possibility that the upper specification limit will be exceeded increases as well in time. Hence, we rather keep the product in a short using time to ensure the product performs in a normal function. Moreover, when the deterioration rate increases, the probability for the product quality value to fall in an acceptable range is reduced, that is, R_Y^* is

Table 1. Upper and lower process capability limits for component.

j	t_{Lj} [mm]	t_{Uj} [mm]
1	0.014	0.042
2	0.018	0.052
3	0.024	0.072
4	0.009	0.027
5	0.010	0.030

Table 2. Tolerance cost function coefficients.

j	a_j	b_j	c_j
1	475.2707	639.1768	64.6773
2	475.2707	639.1768	64.6773
3	882.1414	2731.5370	70.4506
4	388.3970	485.2209	128.3845
5	388.3970	485.2209	128.3845

reduced.

To help the reader's application in the proposed approach, the relevant steps are illustrated in the following:

Step 1: Provide the information of design drawing and the functionality of product attempts. See the example in Fig. 2.

Step 2: Identify the product quality characteristics of interest and the associated quality values of measurement. Provide the design target, design tolerance, material cost, inspection cost, failure cost, and quality loss coefficient of a product and deterioration rate for a deteriorating component.

Step 3: Identify the quality characteristic of the interested controllable variables and the associated quality value for measurement. Of course, the quality value of product quality characteristic must be related to the quality value of interested controllable variables. Give a feasible range of parameter values and process capability limit of tolerance values for controllable variables.

Step 4: An optimization model with an acceptable reliability value is developed to minimize total cost, including quality loss, failure cost, inspection cost, and tolerance cost, by determining optimal initial settings, process mean, process tolerance, and using time simultaneously.

Step 5: Review the results and have optimal process mean and process tolerance. Finally, sensitivity analysis and model discussions on some decision variables are performed.

Table 3. The values of TC^*, Q^*, and R_Y^* versus W_3 and W_4.

W_3 W_4	0.010	0.020	0.030
0.025	TC^* = \$609.2994 Q^* = 5.6696 R_Y^* = 0.9942	TC^* = \$769.1101 Q^* = 4.4820 R_Y^* = 0.9922	TC^* = \$926.5353 Q^* = 3.7173 R_Y^* = 0.9902
0.035	TC^* = \$769.1101 Q^* = 4.4820 R_Y^* = 0.9922	TC^* = \$926.5353 Q^* = 3.7173 R_Y^* = 0.9901	TC^* = \$1081.9500 Q^* = 3.1829 R_Y^* = 0.9880
0.045	TC^* = \$926.5353 Q^* = 3.7173 R_Y^* = 0.9901	TC^* = \$1081.9500 Q^* = 3.1829 R_Y^* = 0.9880	TC^* = \$1235.6090 Q^* = 2.7880 R_Y^* = 0.9857

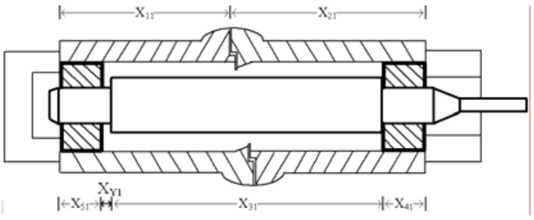

Fig. 2. A shaft-bearing system.

5. Conclusions

The results indicate that the initial setting, process mean, process tolerance, and using time should be optimized simultaneously when deterioration exists in components for a product. The optimal solutions can be obtained from usually accessible software such as GAMS. In other words, the proposed model can be applied in very user-friendly fashion for most product designers. In addition, the optimal solutions determined with the proposed model can achieve a significant reduction in the cost and improvement in the quality of a product. Namely, the product can be used in quality function for a longer life than expected and product failure that leads to expensive and costly payments can be avoided. These achievements can raise the capability in facing intensive competition around today's manufacturing sectors.

Nomenclature

X : Quality value for product quality characteristic of interest

T : Design target of the quality characteristic X

L_1 : Lower design tolerance (design specification) of quality characteristic X

L_2 : Upper design tolerance (design specification) of quality characteristic X

U : Process mean of the quality characteristic X

σ^2 : Process variance of the quality characteristic X

t : Process tolerance of the quality characteristic X

$C_M(t)$: Tolerance cost as a function of process tolerance t

a_0 : Initial setting at the beginning of product life

B : A constant value resulting from initial impact at beginning of product life

W : The deterioration rate for a deteriorating component

$L(X)$: The loss function

C_P : Process capability index

$f_d(\cdot)$: A design function representing a functional relationship between the quality values of component/process quality characteristics and the quality values of product quality characteristics

X_{Yr} : The quality value representing r^{th} product quality characteristic of a final product

A_{rj} : The r^{th} quality chain vector which links r^{th} product quality characteristic with j^{th} component/process

X_{jr} : The quality value representing j^{th} component/process quality characteristic which is an element of r^{th} product quality characteristic

t_{Uj} : Upper process capability limit of j^{th} component/process quality characteristic

t_{Lj} : Lower process capability limit of j^{th} component/process quality characteristic

$f(X_{Yr})$: Normal distribution with resultant process mean and resultant process variance

U_{Yr} : Resultant process mean of the r^{th} product quality characteristic X_{Yr}

$\sigma_{Yr}{}^2$: Resultant process variance of the r^{th} product quality characteristic X_{Yr}

t_{Yr} : Resultant process tolerance of the r^{th} product quality characteristic X_{Yr}

T_r : Design target of the r^{th} product quality characteristic X_{Yr}

L_{1r} : Lower design tolerance of the r^{th} product quality characteristic X_{Yr}

L_{2r} : Upper design tolerance of the r^{th} product quality characteristic X_{Yr}

K_{1r} : Lower quality loss coefficient of the r^{th} product quality characteristic X_{Yr}

K_{2r} : Upper quality loss coefficient of the r^{th} product quality characteristic X_{Yr}

C_{1r} : Lower failure cost of the r^{th} product quality characteristic X_{Yr}

C_{2r} : Upper failure cost of the r^{th} product quality characteristic X_{Yr}

M_B : Total numbers of product quality characteristics (quality chains) of interest in a product

M_0 : Total numbers of components/processes in a product

M_r : Related processes for r^{th} product quality characteristic (quality chain)

C_N : Material cost for a product

IC_r : Inspection cost for r^{th} product quality characteristic

R_{min} : The minimal quality requirement

R_Y : The average quality reliability of a product for M_B product quality characteristics (quality chains) in duration Q

$U_{jr}{}^*$: Optimal process mean of j^{th} component/process quality characteristic for r^{th} product quality characteristic

$a_{0j}{}^*$: Initial setting of of j^{th} component/process quality characteristic

t_j^* : Optimal process tolerance of j^{th} component/process quality characteristic

Q^* : Optimal using time for the product

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Chien-Ping Chung received his B. S., M. S., and Ph. D. degrees in Industrial Engineering from Feng Chia University (Taiwan, ROC). Currently, he is a lecturer at the Department of Industrial Engineering and Systems Management at Feng Chia University, Taiwan. His research interests include Robust Design, Statistical Quality Control, and Design of Experiment.

Angus Jeang received his B.S. degree in Industrial Engineering from Chun Yuan University (Taiwan, ROC) and received his M.S. degree in Industrial Engineering from Kansas State University (USA). He was then employed by the Nuclear Division of Siemens Gammasonics, Inc. in Desplaines, Illinois (USA), serving as a manufacturing engineer. After he received his Ph.D. in Industrial and Manufacturing Engineering from Wayne State University, Detroit, Michigan (USA), he got a teaching position at the Department of Industrial Engineering and Systems Management at Feng Chia University, Taiwan. Then, he acted as head of department for a period of time. Currently, he is a full professor of this department.